Electrical Power and Energy Systems 31 (2009) 482–488

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

A fair method for centralized optimization of multi-TSO power systems

Yannick Phulpin [a,*], Miroslav Begovic [b], Marc Petit [a], Damien Ernst [c]

[a] SUPELEC, Department of Power and Energy Systems, 3 rue Joliot Curie, 91192 Gif sur Yvette, France
[b] Georgia Institute of Technology, Atlanta, GA, USA
[c] University of Liège, Liège, Belgium

ARTICLE INFO

Article history:
Received 8 October 2008
Received in revised form 19 February 2009
Accepted 13 March 2009

Keywords:
Multi-area power system
Centralized control
Multi-party optimization
Multi-objective optimization
Fairness

ABSTRACT

This paper addresses the problem of centralized optimization of an interconnected power system partitioned into several regions controlled by different transmission system operators (TSOs). It is assumed that those utilities have agreed to transferring some of their competencies to a centralized control center, which is in charge of setting the control variables in the entire system to satisfy every utility's individual objective. This paper proposes an objective method for centralized optimization of such multi-TSO power systems, which relies on the assumption that each TSO has a real-valued optimization function focusing on its control area only. This method is illustrated on the IEEE 118 bus system partitioned into three TSOs. It is applied to the optimal reactive power dispatch problem, where the control variables are the voltage settings for generators and compensators. After showing that the method has some properties of fairness, namely freedom from envy, efficiency, accountability, and altruism, we emphasize its robustness with respect to certain biased behavior of the different TSOs.

1. Introduction

Several large-scale disturbances emphasized that secure operation of interconnected power systems requires coordination between transmission system operators (TSOs). In particular, Refs. [1,2] report two major disturbances whose consequences were leveraged by the lack of coordination between the TSOs.

To address the problem of coordination in multi-TSO systems, two main trends have arisen [3,4]. On one hand, ad-hoc decentralized control schemes have been proposed, as in [5,6] for example. On the other hand, efforts have been made to create higher-level entities, which would be in charge of coordinating operation over very large geographical areas. By way of example, the potential benefits of a centralized control center (CCC) to make decisions for multi-TSO systems are emphasized in [7].

It is, however, expected that, even with the creation of a CCC, every TSO will preserve some prerogatives of its own system operation. More specifically, operational objectives are likely to remain defined by the TSOs since they may be influenced by local topology, system architecture, generation capacity, or reliance on traditional engineering practices [8]. In this context, prior to agreeing to transferring some of their competencies to a higher decision level, the TSOs may require some guarantees regarding the fulfillment of their own objectives by the CCC, which may be a conflicting issue, as satisfying the objective of a single TSO may adversely affect other TSOs. As introduced in [9], this ability of the optimization scheme to satisfy the needs of every party is equivalent to the fairness of the optimization scheme.

Negotiation is usually advocated to reach a fair solution for multi-party resource allocation problems [10]. In the case of multi-TSO operation issues, as the optimization scheme should handle short-term operation, negotiation can not be considered a suitable solution. However, the choice of a multi-TSO optimization procedure that would satisfy every party may be subjected to negotiation between the different TSOs.

Several procedures have already been proposed for multi-objective optimization. Most are based on a weighting of the different objectives, as in [11,12], for example, or on a priority given to the objectives, as in [13], where some of the objectives are considered as constraints in the optimization procedure. Those strategies may, however, not be acceptable for every party as they depend on an arbitrary prioritization between the different objectives. Other methods are based on iterative choices made by a central entity, as in [14]. This strategy is also inappropriate for multi-TSO operation, as the arbitrary choice at each iteration could be questioned by the different parties.

In this paper, we propose a new scheme, that could be used by the CCC to solve multi-party optimization problems, when the objective of every TSO can be represented by a real-valued function. The scheme relies on the formulation of the problem as a multi-objective optimization problem and picks a solution that could, at least in principle, bring consensus among the different TSOs. Indeed, besides the fact that the solution minimizes a specific

* Corresponding author.
E-mail address: yannick.phulpin@supelec.fr (Y. Phulpin).

0142-0615/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijepes.2009.03.014

distance from the utopian minimum in a normalized multi-dimensional space, we show that it has some properties of fairness. In addition, we also show that the scheme is robust with respect to certain biased behavior by the different parties.

The paper is organized as follows: Section 2 defines the multi-objective problem and presents the illustrative example. In Section 3, we propose a normalization of the multi-objective problem and a procedure for identifying the best solution in the normalized space. Section 4 shows that the scheme has some properties of fairness in the sense of economics, while Section 5 analyzes certain biased behaviors, which can be adopted by the TSOs to turn the optimization scheme in their favor. Finally, some opportunities for further research are outlined in Section 6.

2. Centralized optimization of a multi-TSO system

In the first part of this section, we describe the multi-objective problem faced by the CCC, and introduce some useful notations. Afterward, the illustrative example is presented.

2.1. Formulation of the problem

We focus on a system partitioned into *NbTSO* areas, where each area i is controlled by a single TSO_i. Each TSO_i is assumed to have its own objective expressed by an objective function $C_i(\mathbf{u})$, where $\mathbf{u}$ is the vector of control variables for the entire system. Although an objective function may be of non-economic nature, it will also be referred to hereafter as "cost function."

In this paper, we assume the existence of a CCC that aims to assess the optimal control vector $\mathbf{u}^*$, which is defined as the solution of the following constrained multi-objective optimization problem

$$\min_{\mathbf{u}}[C_1(\mathbf{u}), C_2(\mathbf{u}), \ldots, C_{NbTSO}(\mathbf{u})] \quad (1)$$

subject to

$$g(\mathbf{u}) = 0 \quad (2)$$
$$h(\mathbf{u}) \leqslant 0 \quad (3)$$

where $g(\mathbf{u}) = 0$ represents the equality constraints, and $h(\mathbf{u}) \leqslant 0$ represents the inequality constraints. From now on, the set of solutions $\mathbf{u}$ such that $\mathbf{u}$ verifies Equality (2) and Inequality (3) will be referred to as U.

The cost associated with a control vector $\mathbf{u}$ can be represented in a *NbTSO*-dimensional cost space by a vector $[C_1(\mathbf{u}), C_2(\mathbf{u}), \ldots, C_{NbTSO}(\mathbf{u})]$. A solution $\mathbf{u}_n$ is said "non-dominated" if there exists no other solution $\mathbf{u} \in U$ such that $\forall i \in [1, 2, \ldots, NbTSO]$, $C_i(\mathbf{u}) \leqslant C_i(\mathbf{u_n})$. In the *NbTSO*-dimensional cost space, the set of non-dominated solutions represents the Pareto-front of the multi-objective problem. It is commonly adopted in the multi-objective optimization literature that the solution of such a problem should be on or as close as possible to its Pareto-front [15].

Consequently, should the Pareto-front be reduced to a single element, the solution of the arbitrage made by the CCC would then be this element. Indeed, in such a particular context, there would exist a solution that minimizes every single objective at once. However, in general, the Pareto-front is composed of many elements (possibly an infinite number of them) and the CCC must choose one of those elements. In this paper, the arbitrage problem to which the CCC is confronted is therefore the problem of selection of the fairest solution on the Pareto-front.

2.2. Illustrative example

As a benchmark system, we use the IEEE 118 bus system partitioned into three areas, referred to as TSO_1, TSO_2, and TSO_3. This system is presented in Fig. 1.

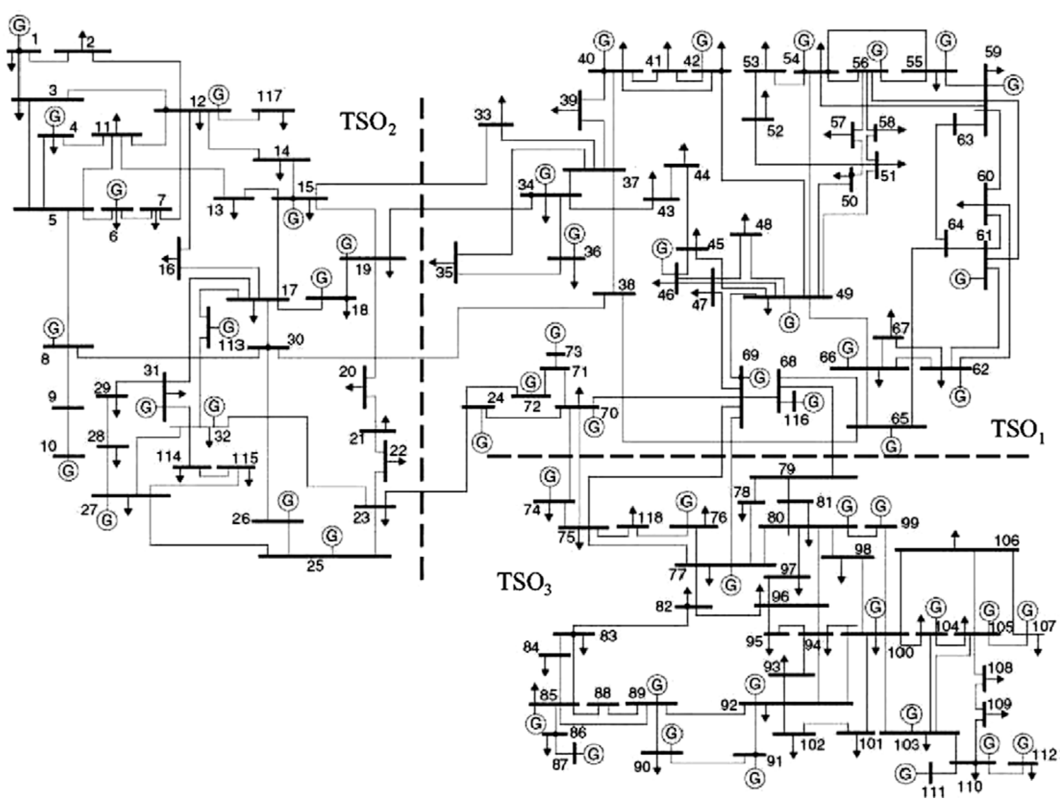

Fig. 1. IEEE 118 bus system with three TSOs.

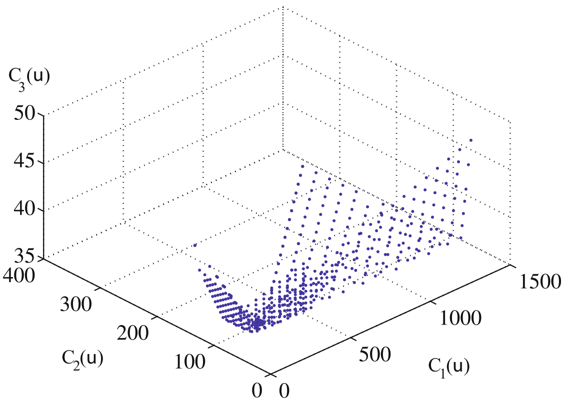

Fig. 2. Representation of the Pareto-front for the IEEE 118 bus system with three TSOs. $C_1(\mathbf{u})$, the objective function of TSO_1, represents reactive power support in area 1, $C_2(\mathbf{u})$ is a weighted function of active power losses and reactive power support in area 2, and $C_3(\mathbf{u})$ represents active power losses in area 3.

The proposed methodology is applied to a multi-TSO reactive power dispatch problem, which is a particular type of optimal power flow. In such a context, the load demand and the active power generation pattern are considered known. We use a decentralized slack bus in our simulations, which may slightly change the generation dispatch depending on active power losses. Every element $\mathbf{u} \in U$ is composed of the generators' output voltages, the capacitor banks' or FACTS' reactive power injections and the tap settings. To model a real system, some of those control actions should be discrete variables, as in [16], for example. However, as the use of discrete and continuous variables would result in a mixed-integer non-linear programming problem, whose solution may be difficult to compute, we have considered only continuous control variables in our simulations.

Equality (2) represents the load flow equations and constant active power export, while Inequality (3) corresponds to the limits on voltage magnitude at each bus as well as reactive power injections for every generator or compensator.

In real systems, every TSO_i may have an objective of a different nature. We consider in this paper three types of cost function. Those are the minimization of active power losses P_i^{Losses}, the minimization of a quadratic sum of reactive power injections ($\sum_{j \in TSO_i} Q_j^2$), and a linear combination of P_i^{Losses} and $\sum_{j \in TSO_i} Q_j^2$. Such formulations of the objectives are commonly used in the literature to describe operational objectives of TSOs (see for example [16–19]) but others, like transmission capacity or voltage stability margin, could also be used [20,21]. Those three types of cost function can be represented by the following objective function

$$C_i(\mathbf{u}) = \gamma_i P_i^{Losses}(\mathbf{u}) + (1 - \gamma_i) \sum_{j \in TSO_i} Q_j^2(\mathbf{u}) \tag{4}$$

where $\gamma_i \in [0, 1]$ is the weight coefficient for area *i*. In our example, γ_1 is equal to 0, γ_2 to 0.9 and γ_3 to 1. The Pareto-front of our illustrative case is represented in Fig. 2.

3. A new method to solve multi-party optimization problems

In this section, we propose an approach for electing the point on the Pareto-front that could satisfy the different parties. We have designed the optimization procedure as follows: first, we suppose that every TSO_i provides the CCC with its objective and constraint functions $\hat{C}_i(\mathbf{u}_{TSO_i})$, $\hat{g}_i(\mathbf{u}_{TSO_i})$, and $\hat{h}_i(\mathbf{u}_{TSO_i})$, where $\mathbf{u}_{TSO_i}$ represents the control variables for TSO_i's area. The symbol ^ on C_i, g_i, and h_i specifies that, since a TSO_i does not systematically know the system topology, generation pattern, and load demand in the other areas, it can only formulate its own objective and constraints as functions of its own system state, defined by $\mathbf{u}_{TSO_i}$. After receiving the information from every TSO_i on its objective and constraint functions $\hat{C}_i(\mathbf{u}_{TSO_i})$, $\hat{g}_i(\mathbf{u}_{TSO_i})$, and $\hat{h}_i(\mathbf{u}_{TSO_i})$, the CCC defines the multi-objective problem ($C_i(\mathbf{u})\ \forall i \in [1, 2, \ldots, NbTSO]$, $g(\mathbf{u})$, and $h(\mathbf{u})$), and, afterward, faces the problem of electing the fairest solution on its Pareto-front.

The proposed approach relies on finding a solution as close as possible to the "utopian minimum" C^{ut} defined in [22] as

$$C^{ut} = [C_1(\mathbf{u}_1^*), C_2(\mathbf{u}_2^*), \ldots, C_{NbTSO}(\mathbf{u}_{NbTSO}^*)] \tag{5}$$

where $\mathbf{u}_i^*$ is the solution of Problem (6), which optimizes the entire system with the unique objective $C_i(\mathbf{u})$ under constraints (2)–(3), that is

$$\mathbf{u}_i^* = \arg\min_{\mathbf{u} \in U} C_i(\mathbf{u}) \tag{6}$$

The approach is based on the following principle: should a "utopian minimum" exist, it would then be chosen as the solution since everyone of TSOs' objectives are minimized with that solution. However, we know that, except if the Pareto-front is reduced to a single element, there is no $\mathbf{u} \in U$ that corresponds to the "utopian minimum." Therefore, we choose the solution $\mathbf{u} \in U$ that minimizes the distance – related to an Euclidean norm after having normalized the cost functions – with the "utopian minimum."

The procedure for normalizing the cost functions is presented in Section 3.1. Section 3.2 describes the procedure for computing the solution that is closest to the utopian minimum in the normalized space. Finally, the approach is illustrated in Section 4 on the IEEE 118 bus system partitioned into three TSOs.

3.1. Normalization

We explain hereafter the normalization process that can be adopted to obtain a fair arbitrage. Its rationale is twofold. First, every local objective can have a different nature (e.g., minimization of active power losses, maximization of reactive power reserves, etc.). This problem should naturally be addressed by the normalization process. Second, it also makes sense to normalize the cost functions to penalize the TSOs whose objective fulfillment is detrimental to other TSOs' objectives and favor those whose objectives are particularly compatible with the others.

For a particular cost function $C_i(\mathbf{u})$, the normalization factor will be the product of the two terms C_i° and χ_i. The normalized cost function $\overline{C_i}(\mathbf{u})$ will thus be computed using the following equation

$$\overline{C_i}(\mathbf{u}) = \frac{C_i(\mathbf{u})}{C_i^\circ \times \chi_i} \tag{7}$$

We note that – since we will pick after normalization a solution that stands closest to the utopian minimum – a small normalization factor for TSO_i will have for effect to give more weight to its own objective function $C_i(\mathbf{u})$ and will then favor it.

The term C_i° is defined as follows:

$$C_i^\circ = \sum_{j=1}^{NbTSO} \frac{C_i(\mathbf{u}_j^*) - C_i(\mathbf{u}_i^*)}{NbTSO} \tag{8}$$

It has been introduced for two main reasons. First, it is expressed in the same unit as $C_i(\mathbf{u})$ and will therefore make possible the comparison between objective functions having different natures. In particular, it will make our approach independent of any scaling factor that may affect the different cost functions $C_i(\mathbf{u})$. Second, the term C_i° will also favor a TSO whose objective fulfillment is weakly penalized by the fulfillment of the other objectives. Indeed, C_i° being the average value of the overcosts[1] supported by TSO_i for

[1] The term "overcosts" refers, in this paper, to the difference between the actual costs $C_i(\mathbf{u})$ and their minimal value $C_i(\mathbf{u}_i^*)$.

the *NbTSO* control variables $\mathbf{u}_1^*, \mathbf{u}_2^*, \ldots, \mathbf{u}_{NbTSO}^*$, this term will be particularly small if the overcosts induced by other objective fulfillments $C_i(\mathbf{u}_j^*)$ are small.

The term χ_i is defined as follows:

$$\chi_i = \sum_{j=1}^{NbTSO} \frac{C_j(\mathbf{u}_i^*) - C_j(\mathbf{u}_j^*)}{C_j^\circ} \quad (9)$$

It has been introduced to penalize the detrimental impact of TSO_i's objective achievement on the other TSOs' costs, represented by the term $C_j(\mathbf{u}_i^*) - C_j(\mathbf{u}_j^*)$. We note that this difference term is divided by C_j°. Thus, this division allows to sum up overcosts having different natures. Also, this normalization aims to leverage the penalization that TSO_i endures when its optimal control variables are detrimental to the objective of another TSO_j, which is itself compatible with the other TSO's objective.

By anticipating the results of Section 4, we find that, by using the normalization factor $C_i^\circ \times \chi_i$, the solution of the arbitrage has some properties of fairness in the economic sense.

3.2. Optimization of the normalized problem

As mentioned earlier, our approach will elect the solution $\mathbf{u}^*$, for which the cost vector $C(\mathbf{u}^*)$ minimizes (in the normalized cost space) the Euclidean distance to the "utopian minimum" under Constraints (2), (3). This problem can be formulated as follows:

$$\mathbf{u}^* = \arg\min_{\mathbf{u}\in U} \sum_{i=1}^{NbTSO} (\overline{C}_i(\mathbf{u}) - \overline{C}_i(\mathbf{u}_i^*))^2 \quad (10)$$

Solving this problem is indeed equivalent to finding the point on the Pareto-front that minimizes the distance to the utopian minimum. As a proof, suppose that $\mathbf{u}^*$ is not on the Pareto-front but solution of (10) under Constraints (2)–(3). Then, there would exist a solution $\mathbf{u}\prime$ such that $C_i(\mathbf{u}\prime) \leqslant C_i(\mathbf{u}^*)$ for every $i \leqslant NbTSO$. In this case, for every area i, we would have $\overline{C}_i(\mathbf{u}\prime) \leqslant \overline{C}_i(\mathbf{u}^*)$ and consequently, $\sum_{i=1}^{NbTSO}(\overline{C}_i(\mathbf{u}\prime) - \overline{C}_i(\mathbf{u}_i^*))^2 \leqslant \sum_{i=1}^{NbTSO}(\overline{C}_i(\mathbf{u}^*) - \overline{C}_i(\mathbf{u}_i^*))^2$. Therefore, $\mathbf{u}^*$ would not be the solution of (10), and the equivalence is proved.

Table 1 summarizes the procedure for computing, according to our strategy, a point on the Pareto-front, which could least displease the different TSOs. This procedure implies solving the optimization problem (10) under Constraints (2)–(3). This problem can be solved using a standard optimal power flow algorithm [23–25].

3.3. Example

The proposed method is illustrated hereafter with the test system described in Section 2.2. Table 2 gives the different costs $C_i(\mathbf{u}_j^*)$, normalized overcosts $\overline{C}_i(\mathbf{u}_j^*) - \overline{C}_i(\mathbf{u}_i^*)$, and terms involved in the computation of the normalization factors. The bottom of the table also gives the costs $C_i(\mathbf{u}^*)$ and normalized overcosts $\overline{C}_i(\mathbf{u}^*) - \overline{C}_i(\mathbf{u}_i^*)$ associated with the solution $\mathbf{u}^*$ of the centralized optimization scheme. In addition, Fig. 3 represents the normalized cost vector corresponding to $\mathbf{u}^*$ and the normalized Pareto-front. As one can observe for this particular example, the solution u^* elected by the scheme is close to the utopian minimum.

Table 1
An algorithm for identifying a fair solution of the multi-objective optimization problem.

Input: For every TSO_i, a real-valued objective function $C_i(\mathbf{u})$ and a constraint vector $\mathbf{g}_i(\mathbf{u})$.
Output: A vector of control variables $\mathbf{u}^*$.
Step 1: For every TSO_i, compute $\mathbf{u}_i^*$, solution of:
$\arg\min_{\mathbf{u}\in U}(\mathbf{u})$ with respect to: $g(\mathbf{u}) \leqslant 0$.
Step 2: Compute the solution $\mathbf{u}^*$ of:
$\arg\min_{\mathbf{u}\in U} \sum_{i=1}^{NbTSO}(\overline{C}_i(\mathbf{u}) - \overline{C}_i(\mathbf{u}_i^*))^2$
with respect to: $g(\mathbf{u}) \leqslant 0$
where $\overline{C}_i(\mathbf{u}) = \frac{C_i(\mathbf{u})}{C_i^\circ \times \chi_i}$
with $C_i^\circ = \sum_j \frac{C_i(\mathbf{u}_j^*) - C_i(\mathbf{u}_i^*)}{NbTSO}$
and $\chi_i = \sum_j \frac{(C_j(\mathbf{u}_i^*) - C_j(\mathbf{u}_j^*))}{C_j^\circ}$.

Table 2
Values of the different costs $C_i(\mathbf{u})$ and normalized overcosts $\overline{C}_i(\mathbf{u}) - \overline{C}_i(\mathbf{u}_i^*)$ for every solution $\mathbf{u}_j^*$ of the single objective optimizations and for the solution $\mathbf{u}^*$ of the centralized decision making scheme. Values of C_i° and χ_i for TSO_i are also reported.

	$i = 1$	$i = 2$	$i = 3$
$C_i(\mathbf{u}_1^*)$	4.36	134.82	44.40
$C_i(\mathbf{u}_2^*)$	1381.00	34.64	47.97
$C_i(\mathbf{u}_3^*)$	1278.28	302.00	37.92
C_i°	883.52	122.51	5.51
χ_i	1.99	3.38	3.62
$\overline{C}_i(\mathbf{u}_1^*) - \overline{C}_i(\mathbf{u}_i^*)$	0	0.2418	0.3245
$\overline{C}_i(\mathbf{u}_2^*) - \overline{C}_i(\mathbf{u}_i^*)$	0.7815	0	0.5032
$\overline{C}_i(\mathbf{u}_3^*) - \overline{C}_i(\mathbf{u}_i^*)$	0.7232	0.6453	0
$C_i(\mathbf{u}^*)$	20.01	37.7	38.13
$\overline{C}_i(\mathbf{u}^*) - \overline{C}_i(\mathbf{u}_i^*)$	0.0089	0.0071	0.0106

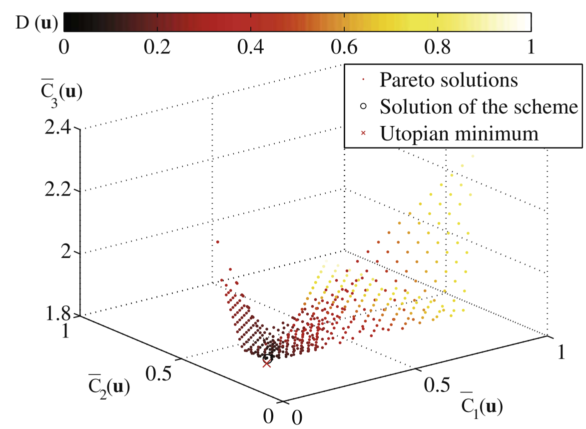

Fig. 3. Localization of the CCC's solution on the normalized Pareto-front for the IEEE 118 bus system partitioned between three TSOs. The color mapping represents the Euclidean distance $D(\mathbf{u})$ (in the normalized cost space) between each solution $\mathbf{u}$ and the utopian minimum. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

4. Fairness of the method

In Section 3, we have presented a new method for choosing a single solution of the multi-objective optimization problem described in Section 2. As introduced in Section 1, this method must have some properties of fairness to be potentially adopted by the TSOs.

The notion of fairness is doubtlessly subjective [26]. Hence, different arbitrages can be simultaneously qualified as fair for any given situation, and a fairness analysis must rely on subjective criteria. As discussed in [27], freedom from envy is an important property of fairness. In addition, the classification proposed by Konow in [28] provides some criteria for assessing the fairness of a particular allocation, namely "efficiency," "accountability," and "altruism." Those criteria have been defined by analyzing experimental data obtained by polling people on their opinions concerning fairness of different types of allocations.

We assess hereafter whether the optimization scheme proposed in Section 3 satisfies those criteria in the context of reactive power dispatch in a multi-TSO system.

4.1. Freedom from envy

As introduced in [27], freedom from envy is a necessary condition of fairness for an allocation scheme. Indeed, an envy-free procedure makes no a priori difference between the different parties, such that no party would prefer to be in the place of another. In practice, all individual objectives of the TSOs must be treated through the same procedure, which must not rely on any specific information on the TSOs.

This is obviously the case with the proposed approach. Indeed, since the procedure has been designed independently of any specific information related to the TSOs, every TSO is equally treated.

4.2. Efficiency

According to Konow, an arbitrage can not be qualified as fair if it is poorly efficient, i.e. if considerable resources are not allocated. While he does not explicitly define the level of efficiency of a given arbitrage for a multi-objective problem, we will consider here that efficiency is maximal if there exists no other arbitrage which can lead to a better outcome for all parties. As suggested in [15], in the case of individual objectives expressed by real-valued functions, the efficiency of an arbitrage may be related to a distance (e.g., the Euclidean distance in the normalized cost space) between an outcome and the Pareto-front of the problem.

In practice, as proved in Section 3.2, the solution of our optimization scheme is on the Pareto-front. Consequently, the elected solution has the property of maximum efficiency, regardless of the objective functions and the constraints.

4.3. Accountability

In the context of multi-party resource allocation, a scheme is accountable if the party investing more effort earns its superior position. An example of an accountable arbitrage is given in [28]: consider two individuals with the same abilities and a global earning that should be divided between them, if one chooses to work 50% less, an accountable notion of fairness would allocate him less earning than the other individual.

Accordingly with the interpretation of accountability proposed in [29], an "effort" of TSO_i could be to make the constraints $g_i(\mathbf{u}) \leqslant 0$ less strict. Let us define, for example, that an effort from one TSO would be the increase of the range of possible bus voltages in its entire control area (say, from $[0.94, 1.06]$ to $[0.92, 1.08]$).

To study the accountability of our arbitrage strategy, we have optimized the base case system with no effort and with an effort from each TSO, successively. Table 3 presents the costs and normalized overcosts supported by each TSO in every case. Those simulation results confirm the observations in [29] that the final allocation is generally more profitable for the TSO that makes more effort, at least in the original cost space. This "accountability" can also be observed in the normalized space, where the overcosts $\overline{C_i}(\mathbf{u}^*) - \overline{C_i}(\mathbf{u}_i^*)$ tend to decrease when TSO_i makes an effort (except for TSO_1 in this example).

However, those observations cannot be generalized since there are some cases for which the final allocation is not accountable. For example, let us consider the case where a TSO_i makes an effort from which it does not directly benefit ($C_i(\mathbf{u}_i^*)$ does not significantly decrease). In such a context, its effort could allow the other TSOs to increase their possible benefits by increasing their use of TSO_i's resources. This could change the normalization factors, especially C_i°, and the location of the utopian minimum, so that the final allocation could be less profitable for TSO_i. In particular, this situation happens when TSO_1 increases the range of possible bus voltages within its control area. For such a case, the decrease of $C_1(\mathbf{u}_1^*)$ is limited, as $\mathbf{u}_1^*$ is not really constrained by the bus voltage limits. In the meantime, $C_1(\mathbf{u}_2^*)$ and $C_1(\mathbf{u}_3^*)$ increase significantly, as the effort made by TSO_1 can be exploited by TSO_2 and TSO_3 in a detrimental way for TSO_1. Consequently, C_1° increases (from 88,352 to 14,692), while χ_1 does not significantly decrease (from 1.99 to 1.56), and the other normalization factors tend to decrease. Hence, despite its higher effort, TSO_1 is penalized ($C_1(\mathbf{u}^*)$ increases from 20.01 to 30.80).

The proposed allocation scheme is also non-accountable if applied to a system with only two parties. The normalization factors for TSO_1 and TSO_2 would then be $C_1^\circ \times \chi_1 = C_1(\mathbf{u}_2^*) - C_1(\mathbf{u}_1^*)$ and $C_2^\circ \times \chi_2 = C_2(\mathbf{u}_1^*) - C_2(\mathbf{u}_2^*)$, respectively. Therefore, one TSO would be rewarded if its objective fulfillment is highly penalizing its neighbor and the arbitrage could not be accountable. This flaw disappears, however, when considering systems with three TSOs or more. Indeed, the more TSOs participate in the process, the more importance is given to a local objective that slightly affects the other TSOs' objectives.

4.4. Altruism

The notion of "altruism" is defined by Konow in [28]. He states that what parties can not influence should not affect the allocation, and proposes the following example of altruism: if two individuals having different abilities work each at 100% of their capabilities, an altruist notion of fairness would allocate them the same share of the global earning. This notion is also developed by Rabin, who associates in [30] the fairness with the concept of "reciprocity."

One property of altruism is that a parameter that does not depend on TSOs' actions should not affect the allocations. The interpretation we make here of this concept is that a optimal control settings for a TSO_i, whose control variables have much influence on the objective fulfillment of the other TSOs, should be consistent with the other TSOs' objectives, regardless of the objective function C^i. However, since the dynamics of the different areas of our benchmark system were highly coupled, we have been unable to check on the test problem whether this concept was indeed satisfied.

Another property of altruism is that the allocations should not be biased toward the TSOs with the greatest "abilities." Indeed, as written in Section 1, the overcosts should rather be shared according to the effort made by the different TSOs. In the context of reactive power scheduling, we consider that the ability of a TSO is related to its influence on the dynamics of the system. Thus, the TSOs that have a strong influence on the system should not have a highly negative impact on the other TSOs. In this respect, the proposed allocation scheme clearly has some altruism proper-

Table 3
Values of the cost functions $C_i(\mathbf{u}^*)$ and normalized overcosts $\overline{C_i}(\mathbf{u}^*) - \overline{C_i}(\mathbf{u}_i^*)$ in every area of the test system. Four cases have been studied: no extra effort, effort from TSO_1, effort from TSO_2, and effort from TSO_3.

Effort	$C_1(\mathbf{u}^*)$	$C_2(\mathbf{u}^*)$	$C_3(\mathbf{u}^*)$
None	20.01	37.70	38.13
TSO_1	30.80	37.02	37.91
TSO_2	15.65	36.71	38.12
TSO_3	19.61	36.95	36.96
	$\overline{C_1}(\mathbf{u}^*) - \overline{C_1}(\mathbf{u}_1^*)$	$\overline{C_2}(\mathbf{u}_2^*) - \overline{C_2}(\mathbf{u}^*)$	$\overline{C_3}(\mathbf{u}^*) - \overline{C_3}(\mathbf{u}_3^*)$
None	0.0089	0.0071	0.0106
TSO_1	0.0125	0.0067	0.0172
TSO_2	0.0087	0.0061	0.0120
TSO_3	0.0070	0.0101	0.0059

ties since the terms χ_i and C_i^0 penalize TSO_i, when its objective is not compatible with the other objectives.

5. Sensitivity to biased information

If a CCC were to apply the proposed resource allocation scheme, some TSOs might be tempted to exercise strategic behavior to turn the scheme in their favor. We discuss in this section how sensitive the optimization scheme is with respect to biased information concerning the constraints (e.g., limitations on voltage or reactive power injections) and objective functions.

5.1. Biased formulation of the constraints

A way for the parties to bias the arbitrage scheme in their favor is to report accountable efforts only. In particular, every TSO_i may be interested to declare more restrictive constraints $g_i(\mathbf{u})$ than those faced in reality, when it does not directly benefit from the relaxation of those constraints. We refer to Section 4.3, for a numerical example of the potential benefits of a TSO, when it provides wrong information about its voltage constraints.

Although the lack of accountability of our scheme with respect to certain types of effort may induce such types of gaming, this non-collaborative strategy might be avoided by continuous monitoring of the power system state by the CCC. For example, a statistical analysis of the bus voltages could inform the CCC about real voltage control abilities of every generator in the power system. The practical implementation of such a policy is, however, particularly complex, and is not discussed in this paper.

5.2. Biased formulation of the objectives

A TSO_i may also be tempted to declare a biased formulation of its cost function. More precisely, a TSO_i could provide the CCC with a function $\widehat{C_i^w}(\mathbf{u}_{TSO_i})$ rather than $\widehat{C}_i(\mathbf{u}_{TSO_i})$.

If $C_i^w(\mathbf{u}) = a \times C_i(\mathbf{u}) + b$ with $a, b \in \Re$, the allocation strategy is not affected since, as emphasized in Section 3.2, our arbitrage strategy has the property of being immune to any linear transformation of the objective functions.[2]

Now, let us consider the case where $C_i^w(\mathbf{u}) = C_i(\mathbf{u}) \times C_i(\mathbf{u})$. Intuitively, with such a biased formulation of its objective function, TSO_i could obtain a better allocation, since it may give to the CCC the impression that a deviation from $\mathbf{u}_i^*$ is worse for it than it is in reality. However, such a strategy is not systematically beneficial for a TSO. For example, if TSO_1, which focuses on the minimization of reactive power support in its control area, asks the CCC to minimize the square of $\sum_{j \in TSO_1} Q_j^2$, the arbitrage leads to a solution were $C_1(\mathbf{u}^*) = 97.07$ rather than 20.01 if TSO_1 were to provide its true objective function. Therefore, such a strategy of overestimating its costs may be counter-productive.

Even if it is clear that, by truncating their objective function, the TSOs might bias the allocation in their favor, such a problem could be avoided in practice by constraining the TSOs to select their cost function in a set of reasonable formulations for the objectives, and report data and constraints truthfully.

6. Conclusions

In this paper, we have addressed the problem of centralized decision making for a multi-TSO power system, for which every TSO's individual objective can be represented by a real-valued cost function. We have emphasized that the problem could be reduced to the election of the fairest point on the Pareto-front. First, we advocated, using "common engineering sense," to select the point which is closest (according to a specific distance measure) to the defined utopian minimum. We have also proposed an algorithm for computing this point. This approach was illustrated on the IEEE 118 bus system, partitioned into three areas having as local objective the minimization of active power losses, reactive power support, or a combination of both criteria. Then, we briefly introduced the concept of fairness in the sense of economics, and we showed that our approach indeed satisfies, at least to some extent, the fairness criteria. We also commented on the robustness of the scheme with respect to some types of strategic behavior from the interconnected TSOs.

Prior to applying the proposed scheme to real systems, some practical issues need to be addressed. In particular, the computational costs of the scheme may be higher than those needed for a single-objective optimal power flow. Indeed, as emphasized in [31], a sophisticated formulation of the objective may induce more computational complexity, which could be critical, when the scheme is applied to large-scale systems. One may also consider other issues in relation to the application of the scheme to real systems. By way of example, with large-scale systems, the individual objectives of the TSOs may be almost independent of a large number of control variables, as those located very far from the area under consideration for example. This could induce a high sensitivity of the normalization factors with respect to some small changes in the system operating conditions, which could be questioned by the different parties.

While the number of potential applications of our method is large (any allocation that can be formulated as a multi-objective problem could be solved through our method), its Achilles' heel is related to the way we define the "fairest allocation" and, more specifically, to the cost functions normalization procedure. This definition is subjective in essence. It may perhaps even be naive to assess the fairness of an allocation without consulting the different parties.

In the framework of multi-TSO power system operation, there is a multitude of tasks, such as dynamic security assessment or transmission investments for which the objective of each party can not be expressed as a real-valued cost function. In such contexts, it would also be interesting, even challenging, to attempt to define the concept of fairest allocation.

Acknowledgement

Damien Ernst is a Research Associate of the Belgian FNRS, from which he acknowledges the financial support.

References

[1] Bialek JW. Are blackouts contagious. IEE Power Eng 2003;17(6):10–3.
[2] Final report – system disturbance on 4 November 2006. In: UCTE; January 2007.
[3] Day LR. Control area trends: principles and responses. IEEE Comp Appl Power 1995;8(2):34–9.
[4] Fardanesh B. Future trends in power system control. IEEE Comp Appl Power 2002;15(3):24–31.
[5] Conejo AJ, Aguado JA. Multi-area coordinated decentralized DC optimal power flow. IEEE Trans Power Syst 1998;13(4):1272–8.
[6] Phulpin Y, Begovic M, Petit M, Heyberger JB, Ernst D. Evaluation of network equivalents for voltage optimization in multi-area power systems. IEEE Trans Power Syst; in press.
[7] Aguado JA, Quintana VH. Inter-utilities power-exchange coordination: a market-oriented approach. IEEE Trans Power Syst 2001;16(3):513–9.
[8] Rebours Y, Kirschen D, Trotignon M, Rossignol S. A survey of frequency and voltage control ancillary services – part I: technical features. IEEE Trans Power Syst 2007;22(1):350–7.
[9] Dubins LE, Spanier EH. How to cut a cake fairly? Amer Math Monthly 1961;68(1):1–17.
[10] Rubenstein-Montano B, Malaga R. A weighted-sum genetic algorithm to support multiple-party multiple-objective negotiations. IEEE Trans Evol Comput 2002;6:366–77.

[2] The independence of the arbitrage with respect to a translation $+b$ is due to the fact that only overcosts are used to define the normalization factors.

[11] Gembicki FW, Haimes YY. Approach to performance and sensitivity multiobjective optimization: the goal attainment method. IEEE Trans Automat Control 1975;20(6):769–71.
[12] Chen YL, Liu CC. Multiobjective VAR planning using the goal-attainment method. IEEE Proc Gener Transm Distrib 1994;141(3):227–32.
[13] Hsiao YT, Chiang HD, Liu CC, Chen YL. A computer package for optimal multi-objective VAR planning in large-scale power systems. IEEE Trans Power Syst 1994;9(2):668–76.
[14] Zionts S, Wallenius J. An interactive programming method for solving the multiple-criteria problem. Manage Sci 1976;22(6):652–63.
[15] Lim LH, Benbasat I. From negotiation to negotiation support systems: a theoretical perspective. In: Proceedings of the HICSS 1992; January 1992. p. 153–63.
[16] Miguelez E, Echavarren Cerezo F, Rodriguez L. On the assignment of voltage control ancillary service of generators in spain. IEEE Trans Power Syst 2007;22(1):367–75.
[17] Janssens N. Tertiary and secondary voltage control for the Belgian HV system. In: Proceedings of the IEE colloquium on international practices in reactive power control; April 1993. p. 81–4.
[18] Deeb N, Shahidehpour SM. Linear reactive power optimization in a large power network using the decomposition approach. IEEE Trans Power Syst 1990;5(2):428–38.
[19] Venkatesh B, Sadasivam G, Khan MA. A new optimal reactive power scheduling method for loss minimization and voltage stability margin maximization using successive multi-objective fuzzy LP technique. IEEE Trans Power Syst 2000;15(2):844–51.
[20] Wu H, Yu CW, Xu N, Lin XJ. An OPF based approach for assessing the minimal reactive power support for generators in deregulated power systems. Int J Elect Power Energy Syst 2008;30(1):23–30.
[21] He R, Taylor GA, Song YH. Multi-objective optimal reactive power flow including voltage security and demand profile classification. Int J Elect Power Energy Syst 2008;30(5):327–36.
[22] Das I, Dennis JE. Normal-boundary intersection: a new method for generating the Pareto surface in nonlinear multicriteria optimization problems. SIAM J Optim 1998;8(3):631–57.
[23] Quintana VH, Torres GL, Medina-Palomo J. Interior-point methods and their applications to power systems: a classification of publications and software codes. IEEE Trans Power Syst 2000;15(1):170–6.
[24] Orfanogianni T, Bacher R. Increased OPF code development efficiency by integration of general purpose optimization and derivative computation tools. In: Proceedings of the 21st IEEE international conference on power industry computer applications, PICA'99; May 1999. p. 123–9.
[25] Capitanescu F, Glavic M, Ernst D, Wehenkel L, Interior-point based algorithms for the solution of optimal power flow problems. Electric Power Syst Res 2007;77(5–6):508–17.
[26] Dickinson DL, Tiefenthaler J. What is fair? Experimental evidence. South Econ J 2002;69(2):414–28.
[27] Feldman A, Kirman A. Fairness and envy. Amer Econ Rev 1974;64:995–1005.
[28] Konow J. A positive theory of economic fairness. J Behav Organization 1996;31:13–35.
[29] Phulpin Y, Begovic M, Petit M, Ernst D. On the fairness of centralised decision-making strategies in multi-TSO power systems. In: Proceedings of the PSCC 2008; July 2008. p. 1–8.
[30] Rabin M. Incorporating fairness into game theory and economics. Amer Econ Rev 1993;83:1281–302.
[31] Wang H, Murillo-Sanchez CE, Zimmerman RD, Thomas RJ. On computational issues of market-based optimal power flow. IEEE Trans Power Syst 2007;22(3):1185–93.